Chapter 1
THE FISHER TRANSFORM

"This is a synopsis of my book," said Tom abstractly

It is commonly assumed that prices have a Gaussian, or Normal, Probability Density Function (PDF). A Gaussian PDF is the familiar bell-shaped curve where 68% of all samples fall within one standard deviation about the mean. This is a really bad assumption, and is the reason many trading indicators fail to produce as expected.

Suppose prices behave as a square wave. If you tried to use the price crossing a moving average as a trading system you would be destined for failure because the price has already switched to the opposite value by the time the movement is detected. There are only two price values. Therefore, the probability distribution is 50% that the price will be at one value or the other. There are no other possibilities. The probability distribution of the square wave is shown in Figure 1. Clearly, this probability function does not remotely resemble a Gaussian probability distribution.

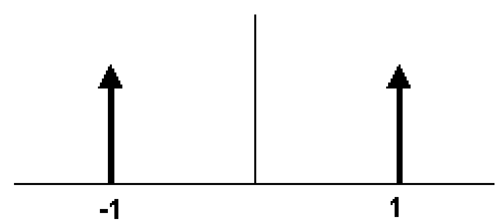


Figure 1.1. The Probability Distribution of a Square Wave Only has Two Values

There is no great mystery about the meaning of a probability density or how it is computed. It is simply the likelihood the price will assume a given value. Think of it this way: Construct any waveform you choose by arranging beads strung on a series of parallel horizontal wires. After the waveform is created, turn the frame so the wires are vertical. All the beads will fall to the bottom, and the number of beads on each wire will stack up to demonstrate the probability of the value represented by each wire.

I used a slightly more sophisticated computer code, but nonetheless the same idea, to create the probability distribution of a sine wave in Figure 2. In this case, I used a total of 10,000 "beads". This PDF may be surprising, but if you stop and think about it, you will realize that most of the sampled data points of a sine wave occur near the maximum and minimum extremes. The PDF of a simple sine wave cycle is not at all similar to a Gaussian PDF. In fact, cycle PDFs are more closely related to those of a square wave. The high probability of a cycle being near the extreme values is one of the reasons why cycles are difficult to trade. About the only way to successfully trade a cycle is to take advantage of the short term coherency and predict the cyclic turning point.

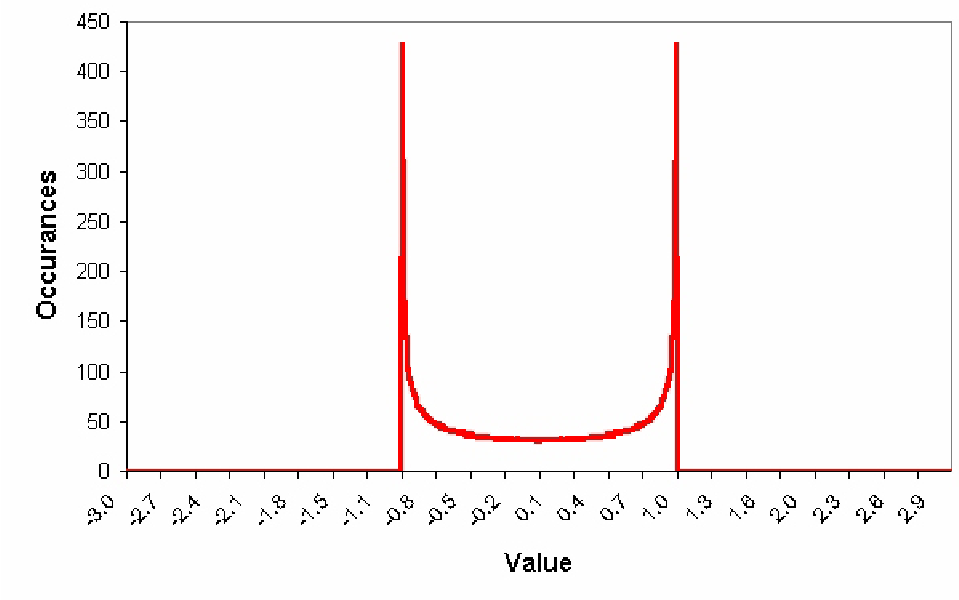


Figure 1.2. Sinewave Cycle Probability Density Function Does Not Resemble a Gaussian Probability Density Function.

The Fisher Transform changes the PDF of any waveform so that the transformed output has an approximately Gaussian PDF. The Fisher Transform equation is:

$$y = .5 * \ln\left[\frac{1+x}{1-x}\right]$$

Where: x is the input
y is the output
ln is the natural logarithm

The transfer function of the Fisher Transform is shown in Figure 1.3.

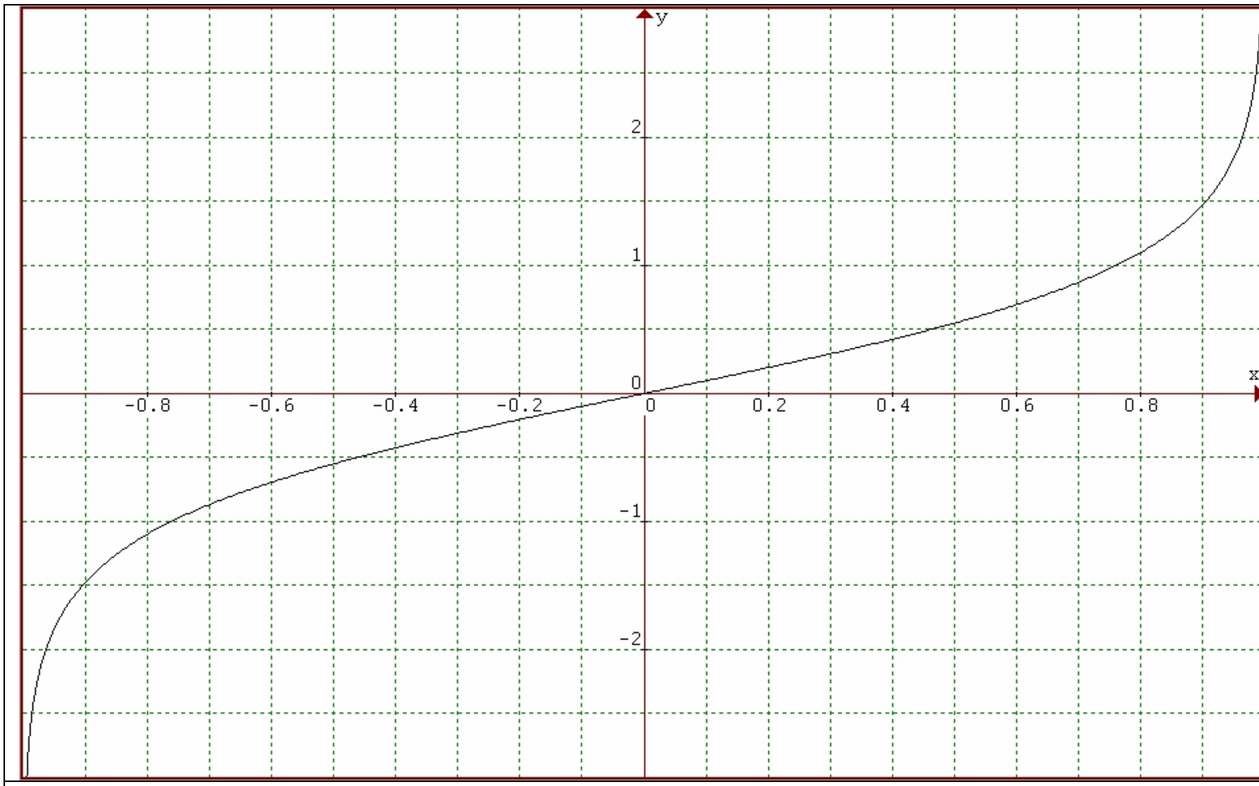


Figure 1.3. The Nonlinear Transfer of the Fisher Transform Converts Inputs (x Axis) to Outputs (y Axis) having a nearly Gaussian Probability Distribution Function

The input values are constrained to be within the range -1 < X < 1. When the input data is near the mean, the gain is approximately unity. For example, go to x = 0.5 in Figure 1.3. There, the Y value is only slightly larger than 0.5. By contrast, when the input approaches either limit within the range the output is greatly amplified. This amplification accentuates the largest deviations from the mean, providing the “tail” of the Gaussian PDF. Figure 1.4 shows the PDF of the Fisher Transformed output as the familiar bell shaped curve, compared to the input sinewave PDF. Both have the same probability at the mean value. The transformed output Probability Density Function is nearly Gaussian, a radical change from the Sinewave PDF.

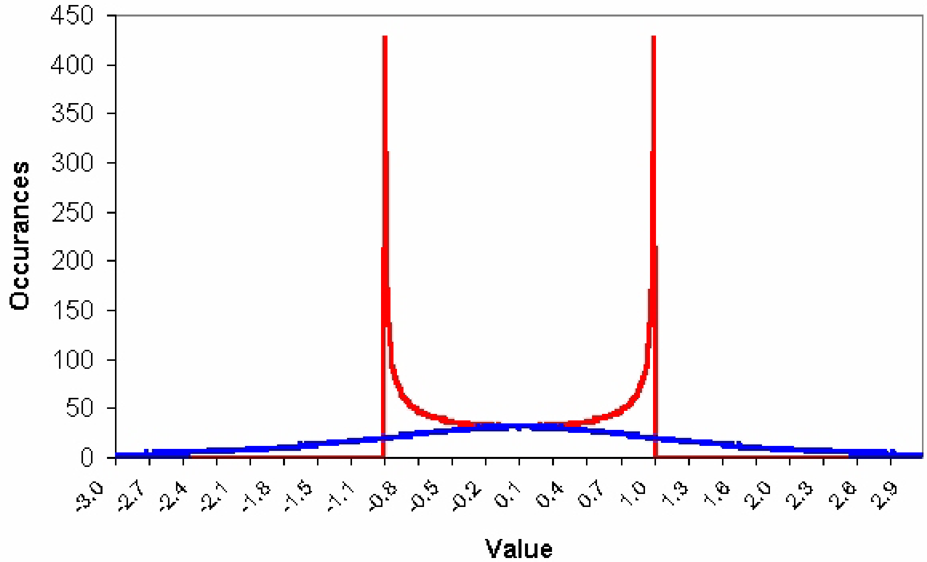


Figure 1.4. The Fisher Transformed Sinewave Has a Nearly Gaussian Probability Density Function Shape

I measured the probability distribution of US Treasury Bond Futures over a 15 year span from 1988 to 2003. To make the measurement I created a normalized channel 10 bars long. The normalized channel is basically the same as a 10 bar Stochastic Indicator. I then measured the price location within that channel in 100 bins and counted up the number of times the price was in each bin. The results of this probability distribution measurement are shown in Figure 1.5. This actual probability distribution more closely resembles the PDF of a sinewave rather than a Gaussian PDF. I then increased the length of the normalized channel to 30 bars to test the hypothesis that the sinewave-like probability distribution is only a short term phenomenon. The resulting probability distribution is shown in Figure 1.6. The probability distributions of Figures 1.5 and 1.6 are very similar. I suspect the analysis can be extended to any market with substantially the same result.

Figure 1.5. Probability Distribution of Treasury Bond Futures
In a 10 bar channel over 15 Years

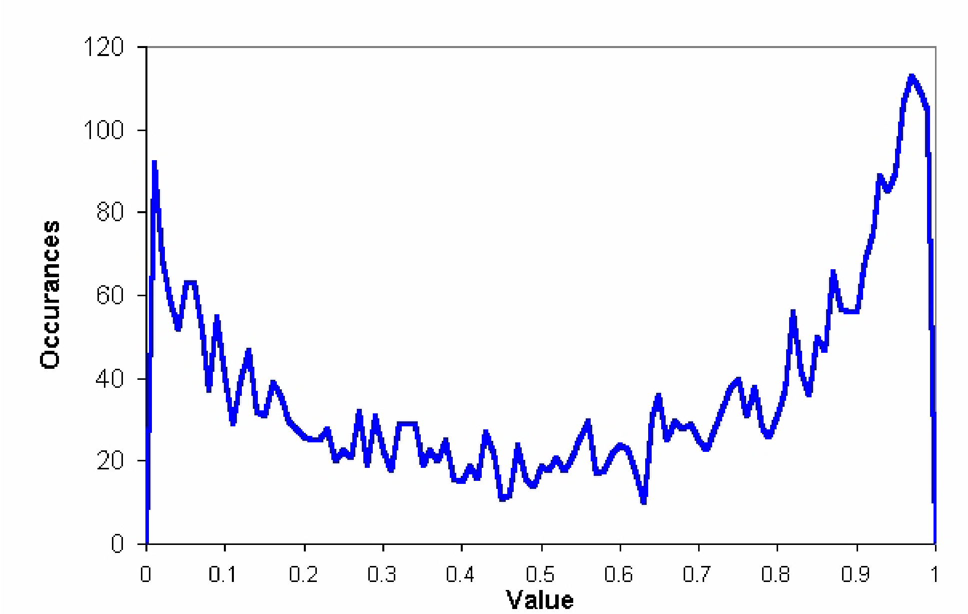
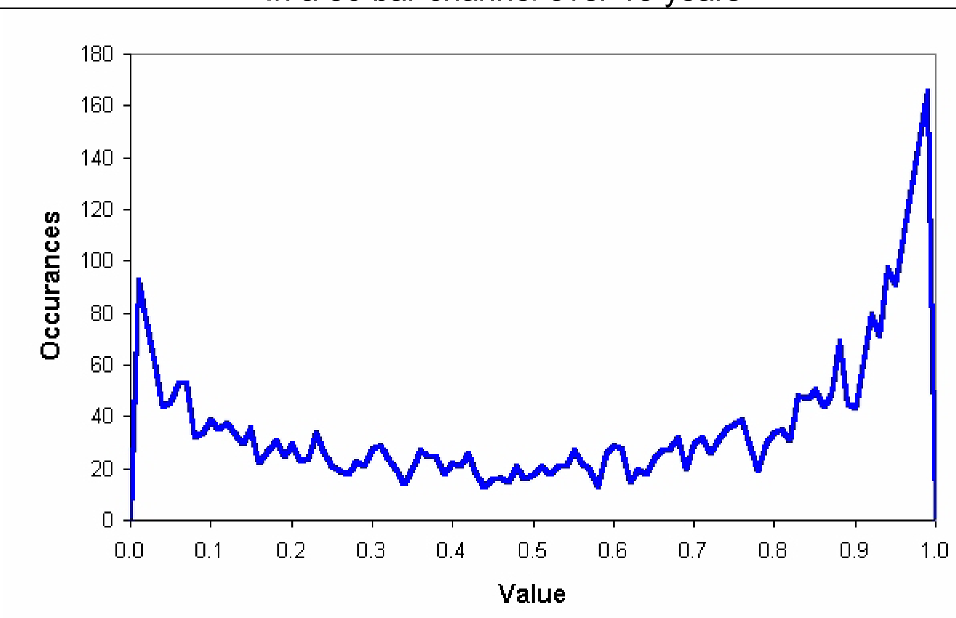


Figure 1.6. Probability Distribution of Treasury Bond Futures
In a 30 bar channel over 15 years



So what does this mean to trading? If the prices are normalized to fall within the range from –1 to +1 and subjected to the Fisher Transform, the extreme price movements are relatively rare events. This means the turning points can be clearly and unambiguously identified. The EasyLanguage code to do this is shown in Figure 1.7. Value1 is a function to normalize price within its last 10 day range. The period for the range is adjustable as an input. Value1 is centered on its midpoint and then doubled so that Value1 will swing between the –1 and +1 limits. Value1 is also smoothed with an exponential moving average whose alpha is 0.5. The smoothing may allow Value1 to exceed the ten day price range, so limits are introduced to preclude the Fisher Transform from blowing up by having an input value larger than unity. The Fisher Transform is computed to be the variable “Fish”. Both Fish and Fish delayed by one bar are plotted to provide a crossover system that identifies the cyclic turning points.

```
Inputs: Price((H+L)/2),
        Len(10);

Vars:   MaxH(0),
        MinL(0),
        Fish(0);

MaxH = Highest(Price, Len);
MinL = Lowest(Price, Len);

Value1 = .5*2*((Price - MinL)/(MaxH - MinL) - .5) + .5*Value1[1];
If Value1 > .999 then Value1 = .999;
If Value1 < -.999 then Value1 = -.999;

Fish = .5*Log((1 + Value1)/(1 - Value1)) + .5*Fish[1];

Plot1(Fish, "Fisher");
Plot2(Fish[1], "Trigger");
```

Figure 1.7. EasyLanguage Code to Normal Price to a Ten Day Channel And Compute Its Fisher Transform

The Fisher Transform of the prices within a 8 day channel is plotted in the first subgraph below the price bars in Figure 1.8. Note that the turning points are not only sharp and distinct, but they occur in a timely fashion so that profitable trades can be entered. The Fisher Transform is also compared to a similarly scaled MACD indicator in subgraph 2 of Figure 1.8. The MACD is representative of conventional indicators whose turning points are rounded and indistinct in comparison to the Fisher Transform. As a result of the rounded turning points, the entry and exit signals are invariably late.

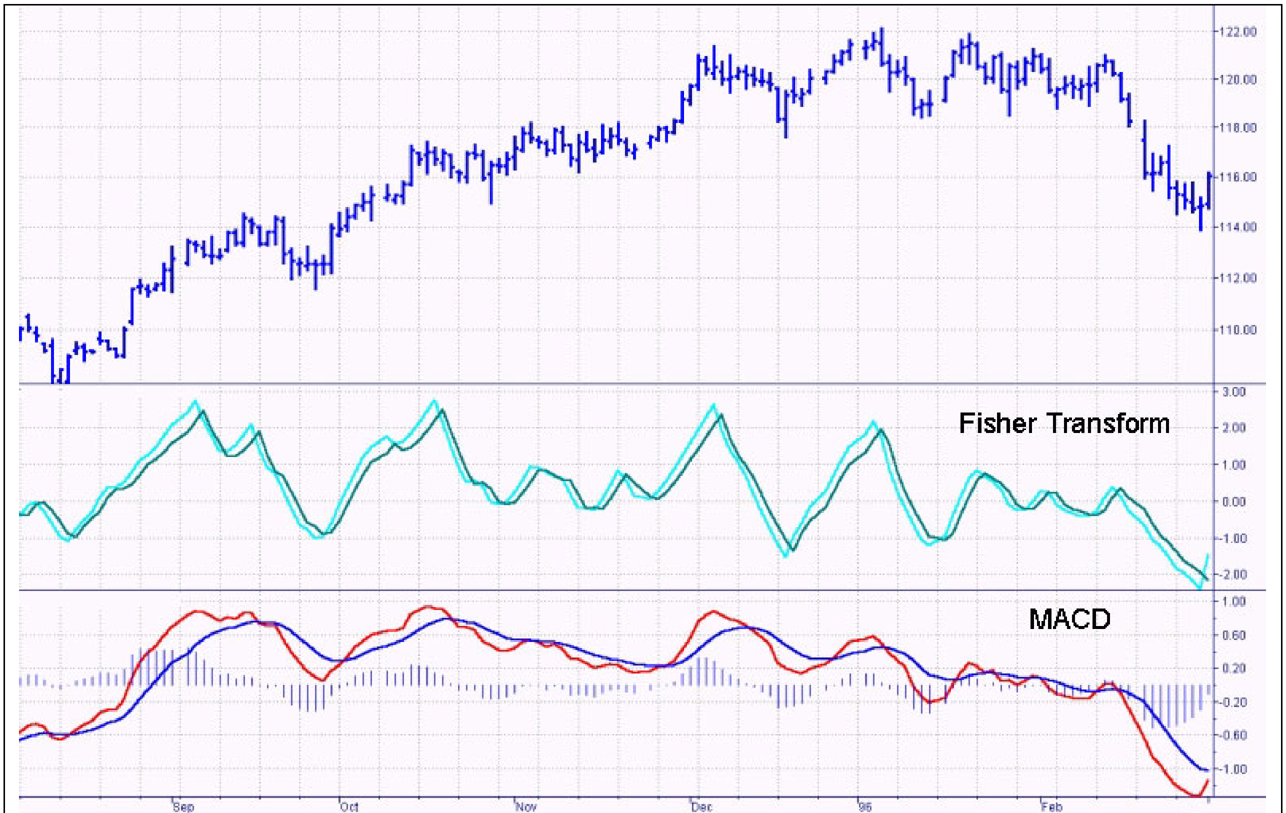


Figure 1.8. The Fisher Transform of Normalized Prices Has Very Sharp Turning Points When Compared to Conventional Indicators such as the MACD

KEY POINTS TO REMEMBER

1) Prices almost always do not have a Gaussian, or Normal, probability distribution.
2) Statistical measures based on Gaussian probability distributions, such as a standard deviation, are in error because the probability distribution assumption underlying the calculation is in error.
3) The Fisher Transform converts almost any input probability distribution to be nearly a Gaussian probability distribution.
4) The Fisher Transform, when applied to indicators, provides razor-sharp buy and sell signals.